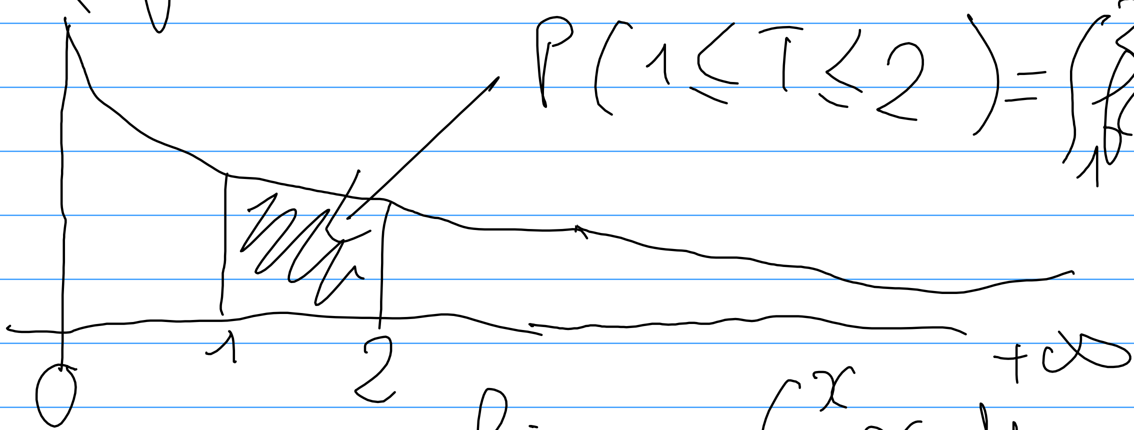
Activité d'introduction
Lois à densité

Fiche

Partie B

Question 1 :

$f(x) = 0{,}2\, e^{-0{,}2x}$ sur $[0;+\infty[$



$P(1 \leq T \leq 2) = \int_1^2 f(x)\,dx$

$\lim\limits_{x \to +\infty} \int_0^x f(t)\,dt = 1$

1) et 2)

$P(0 \leq T \leq 3) = \int_0^3 f(x)\,dx$

$y = f(x)\,dx$

donc $P(0 \leq T \leq 3) = \int_0^3 0{,}2e^{-0{,}2x}\,dx$

$= \left[-e^{-0{,}2x}\right]_0^3$

$= -e^{-0{,}2\times 3} + 1$

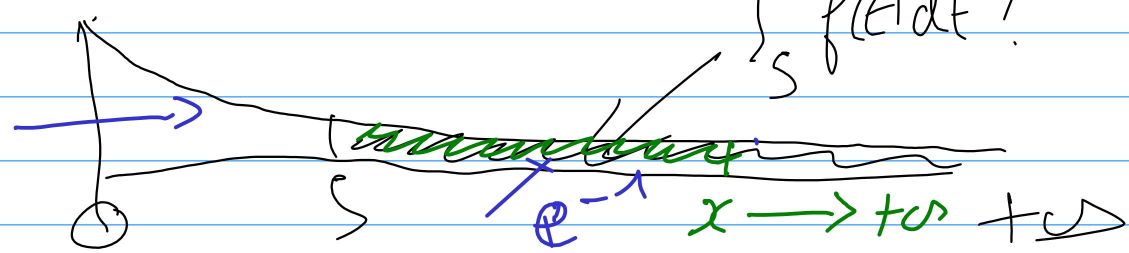
$P(0 \leq T \leq 3) = 1 - \underbrace{e^{-0{,}6}}_{<1} \simeq 0{,}451$

3) « Au moins 5 minutes » $= \{T \geq 5\}$

$$P(T \geq 5) = \lim_{x \to +\infty} \int_5^x f(t)\,dt$$

$$\int_5^{+\infty} f(t)\,dt \ ?$$

$1 - e^{-1}$



On passe plutôt au complémentaire :

$$P(T \geq 5) = 1 - P(T < 5)$$

$$= 1 - \int_0^5 0{,}2e^{-0{,}2t}\,dt$$

$$= 1 - \left[-e^{-0{,}2t}\right]_0^5$$

$$= 1 - \left(-e^{-0{,}2 \times 5} + 1\right)$$

$$\boxed{P(T \geq 5) = e^{-0{,}2 \times 5} = e^{-1}}$$

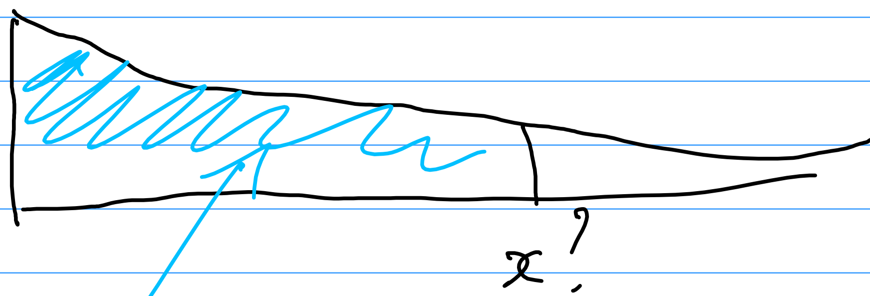
$$\int_5^x f(t)\,dt = \left[-e^{-0{,}2t}\right]_5^x = -e^{-0{,}2x} + e^{-1}$$

Il faut faire tendre x vers $+\infty$

$$\lim_{x \to +\infty} -e^{-0{,}2x} = 0 \quad \text{donc} \quad \lim_{x \to +\infty} \int_0^x f(t) = e^{-1}$$

4) On cherche x tel que

$P(T \leq x) > 0{,}95$



$P(T \leq x)$ soit supérieure à 0,95

On cherche le plus petit x vérifiant cette condition

$$\int_0^x 0{,}2e^{-0{,}2t}\,dt > 0{,}95$$

$$\Longleftrightarrow \left[-e^{-0{,}2t}\right]_0^x > 0{,}95$$

$$\Longleftrightarrow -e^{-0{,}2x} + 1 > 0{,}95$$

$$\Longleftrightarrow -e^{-0{,}2x} > -0{,}05$$

$$\Longleftrightarrow e^{-0{,}2x} < 0{,}05$$

$$\ln(e^{-0{,}2x}) < \ln(0{,}05)$$

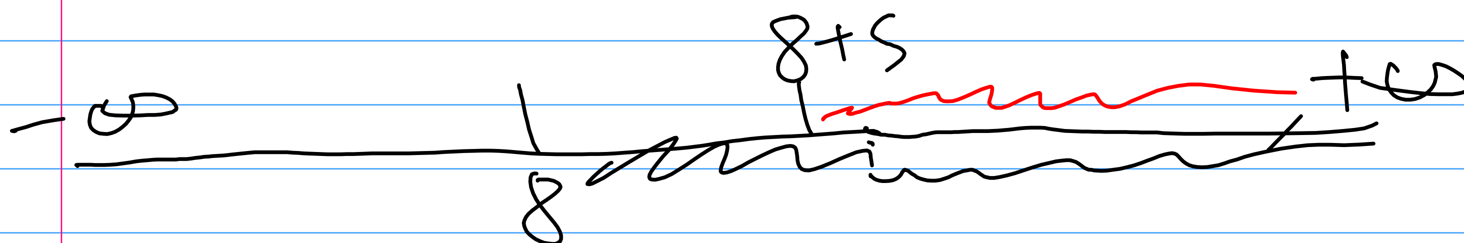
$$\Longleftrightarrow -0{,}2x < \ln(0{,}05)$$

$$\Longleftrightarrow x > \frac{\ln(0{,}05)}{-0{,}02} \approx 15$$

1)

$$P_B(A) = \frac{P(A \cap B)}{P(B)}$$

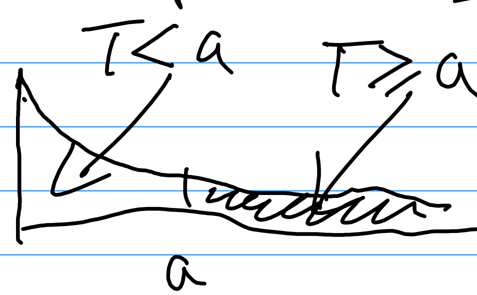
$$P_{(T \geq 8)}(T \geq 8+5) = \frac{P((T \geq 8+5) \cap (T \geq 8))}{P(T \geq 8)}$$



$$(T \geq 8+5) \cap (T \geq 8) = (T \geq 8+5)$$

donc $$P_{(T \geq 8)}(T \geq 8+5) = \frac{P(T \geq 8+5)}{P(T \geq 8)}$$

Soit a un réel ; exprimons $P(T \geq a)$:



$P(T \geq a) = ?$

$$P(T \geq a) = 1 - P(T < a)$$
$$= 1 - \int_0^a 0{,}2e^{-0{,}2t}\,dt$$

$$P(T \geq a) = 1 - \int_0^a 0{,}2e^{-0{,}2t}\,dt$$

$$= 1 - \left[-e^{-0{,}2t}\right]_0^a$$

$$P(T \geq a) = 1 - \left(-e^{-0{,}2a} - (-1)\right)$$

$$\boxed{P(T \geq a) = e^{-0{,}2a}}$$

formule

$$P_{T \geq 8}(T \geq 8+5) = \frac{P(T \geq 8+5)}{P(T \geq 8)}$$

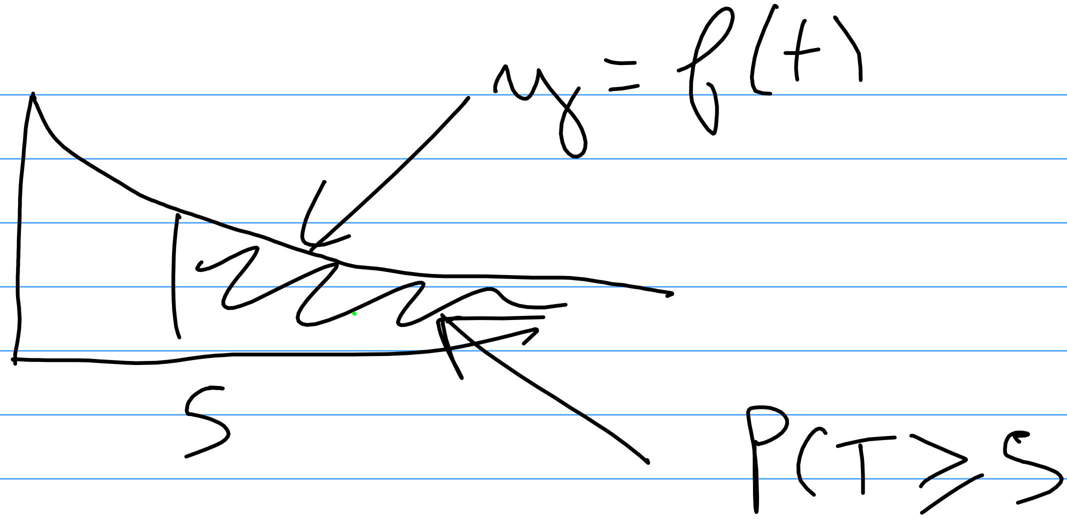
$$= \frac{e^{-0{,}2(8+5)}}{e^{-0{,}2 \times 8}}$$

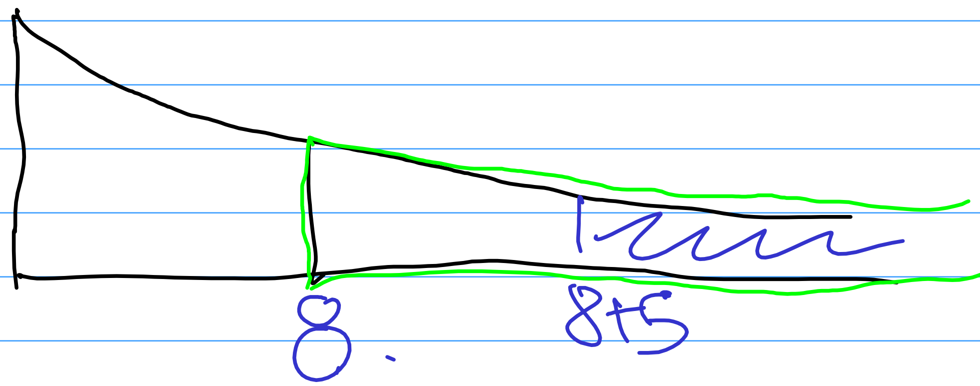
$$\frac{e^{a+b}}{e^a} =$$

$$= e^{-0{,}2(8+5) - (-0{,}2 \times 8)}$$

$$= e^{-0{,}2 \times 5} = e^{-1}$$

$$\boxed{P_{T \geq 8}(T \geq 8+5) = P(T \geq 5)}$$





$$P_{T \geq 8}(T \geq 8+s) = P(T \geq s)$$

Exercice 3 Fiche Calcul Intégral

1) $\displaystyle\int_{10}^{90} \frac{1}{x}\,dx = \Big[\ln(x)\Big]_{10}^{90}$

$= \ln(90) - \ln(10)$

$= \ln\left(\frac{90}{10}\right) = \ln(9)$

$= \ln(3^2) = 2\ln(3)$

2) Primitive de :

$f(x) = \frac{1}{x}\ln(x)$ de la forme $v'(x) \times v(x)$

donc $F(x) = \frac{1}{2}v^2(x) = \frac{1}{2}\ln^2(x)$

3) Valeur moyenne de de f sur $[10;90]$:

$\displaystyle\frac{1}{b-a}\int_a^b f(x)\,dx$

$\displaystyle= \frac{1}{90-10} \times \int_{10}^{90} \frac{500(\ln(x)-2)}{x}\,dx$

On utilise la linéarité :

$$\frac{1}{90-10} \times \left(\int_{10}^{90} 500 \times \frac{\ln(x)}{x} dx - \int_{10}^{90} \frac{2}{x} dx \right)$$

$$= \frac{1}{80} \times \left(500 \times \int_{10}^{90} \frac{\ln(x)}{x} dx - 2 \times \int_{10}^{90} \frac{1}{x} dx \right)$$

$$= \frac{1}{80} \times \left(500 \times \left[\frac{1}{2} \ln^2(x) \right]_{10}^{90} - 2 \ln(9) \right)$$