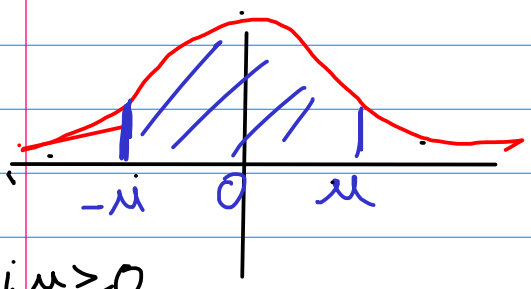
Loi normale $\mathcal{N}(0;1)$

Propriétés de symétrie les plus importantes

- $P(X \leq 0) = P(X \geq 0) = 0{,}5$
- Si $u \geq 0$:



$P(-u \leq X \leq u) = 2P(0 \leq X \leq u)$
$= 1 - 2P(X \geq u)$

Si $u \geq 0$

$P(X \leq u) = P(X \leq -u) + P(-u \leq X \leq u)$
$= \frac{1 - P(-u \leq X \leq u)}{2} + P(-u \leq X \leq u)$

$$P(X \leq u) = \frac{1 + P(-u \leq X \leq u)}{2}$$

$$2P(X \leq u) - 1 = P(-u \leq X \leq u)$$

Cours Exemple 7 :

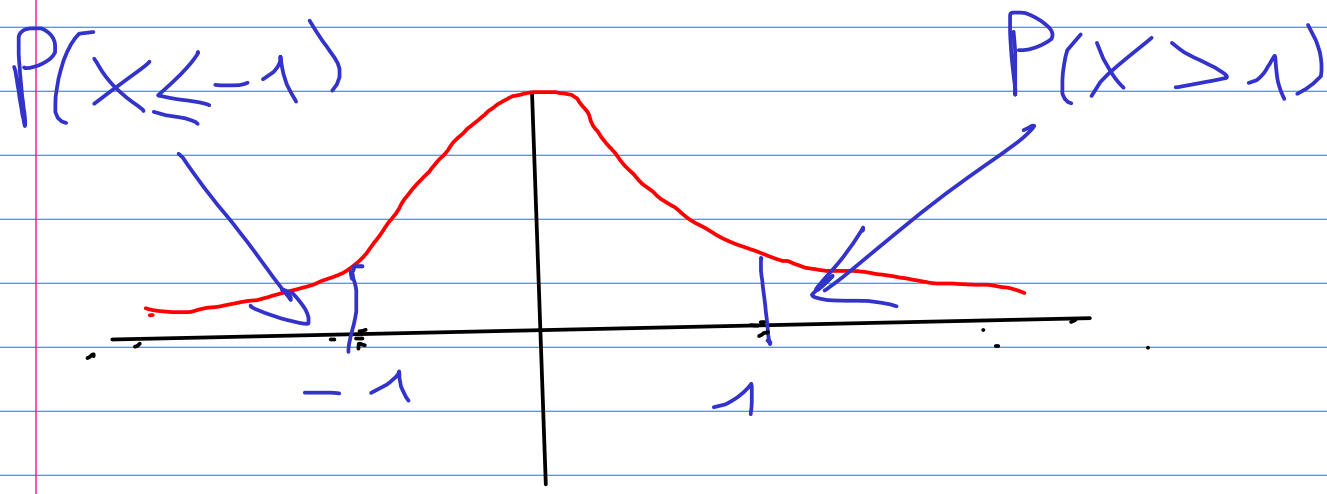
- X suit la loi $\mathcal{N}(0;1)$
$\mu \quad \sigma^2$

On donne $P(X \leq 1) \approx 0,841$

et $P(X \leq -2) \approx 0,023$

$P(X > 1) = 1 - P(X \leq 1)$

$\approx 0,159$



$P(X \leq -1) = P(X < -1) = P(X > 1)$

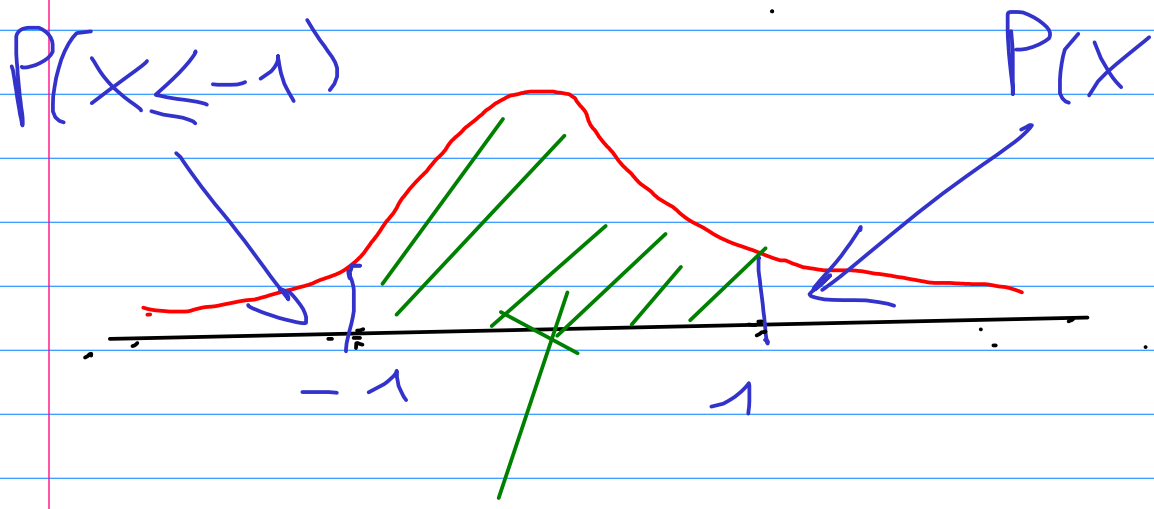
par symétrie

$P(X \geq -1) = 1 - P(X < -1)$

$\approx 0,841$

$$P(X=1) = \int_1^1 \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

$$P(X=1) = 0$$



$$P(-1 \leq X \leq 1)$$

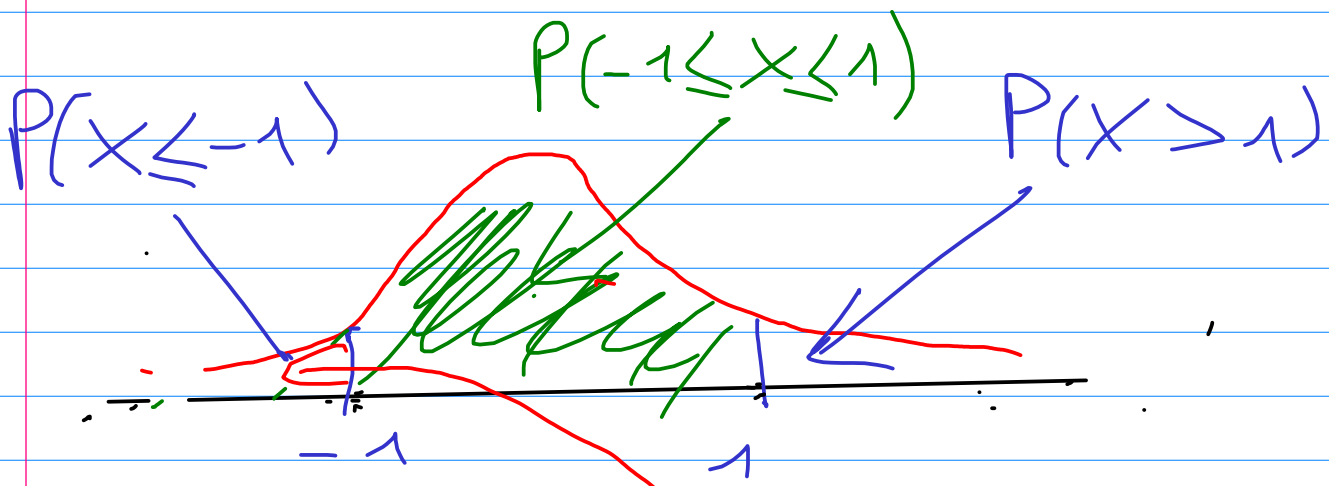
$$P(-1 \leq X \leq 1) = 1 - P(X>1) - P(X<-1)$$

$$= 1 - 2P(X>1)$$

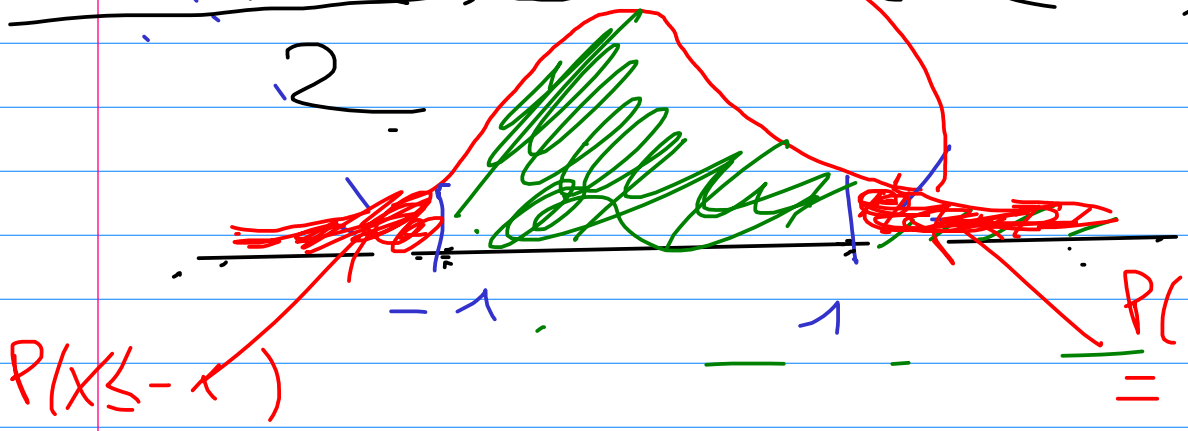
$$\simeq 1 - 2 \times 0,159$$

$$\simeq 0,682$$

$$P(-1 \leq X \leq 1) = 2 \times P(X < 1) - 1$$
$$\approx 2 \times 0{,}841 - 1$$
$$\approx 0{,}682$$



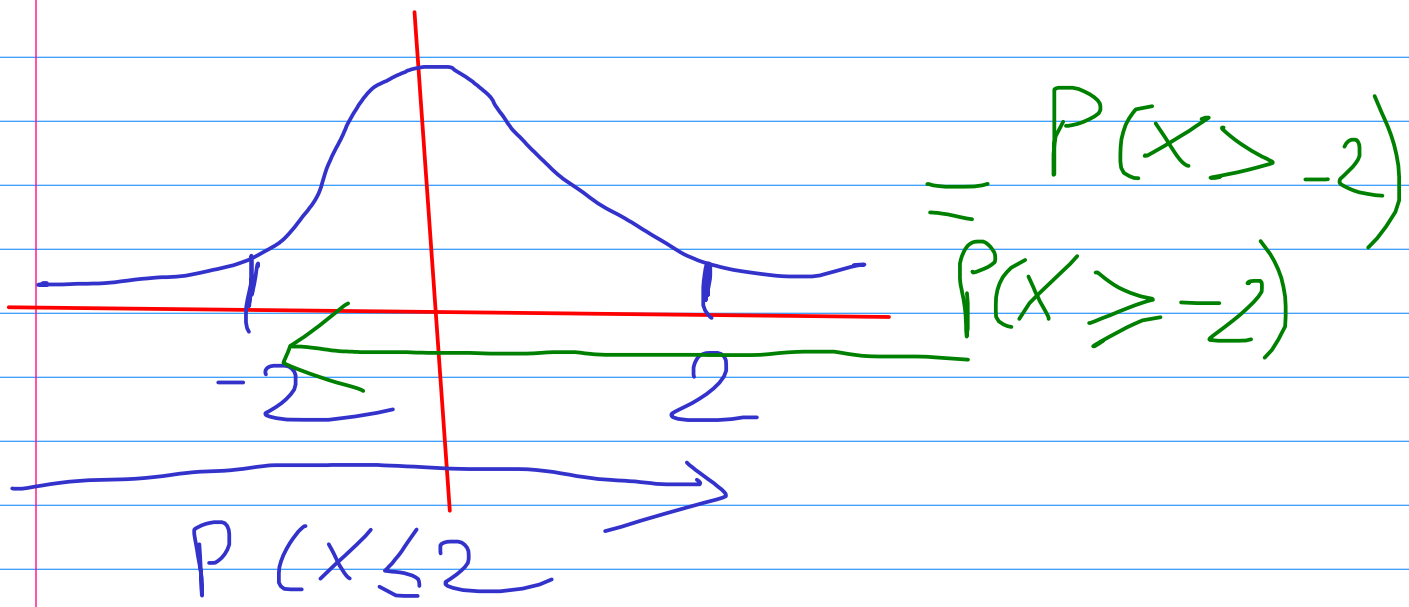
$$\frac{1 + P(-1 \leq X \leq 1)}{2} = P(X \leq 1)$$



$$P(-2 < X \leq 1) = P(X < 1) - P(X \leq -2)$$
$$\approx 0{,}841 - 0{,}023$$
$$\approx 0{,}818$$

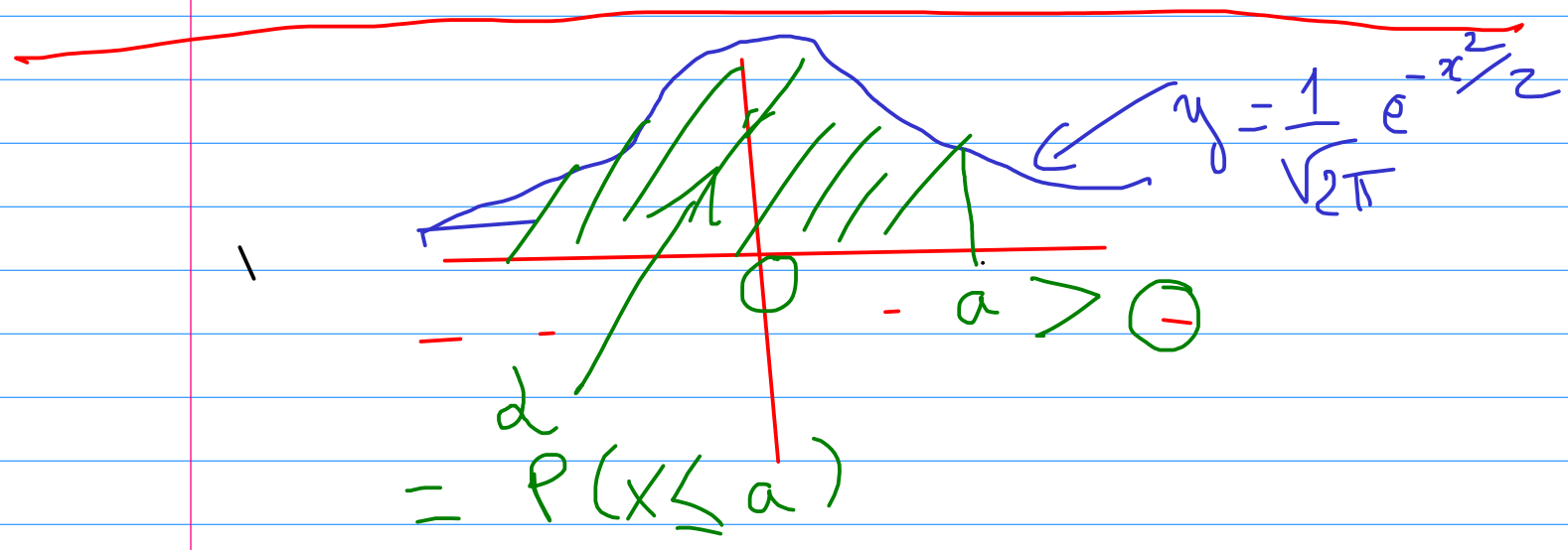
$$P(-2 < X) = 1 - P(X \leq -2)$$

$$\approx 0{,}977$$



$$P(X \leq 2) = P(-2 < X) \approx 0{,}977$$

par symétrie



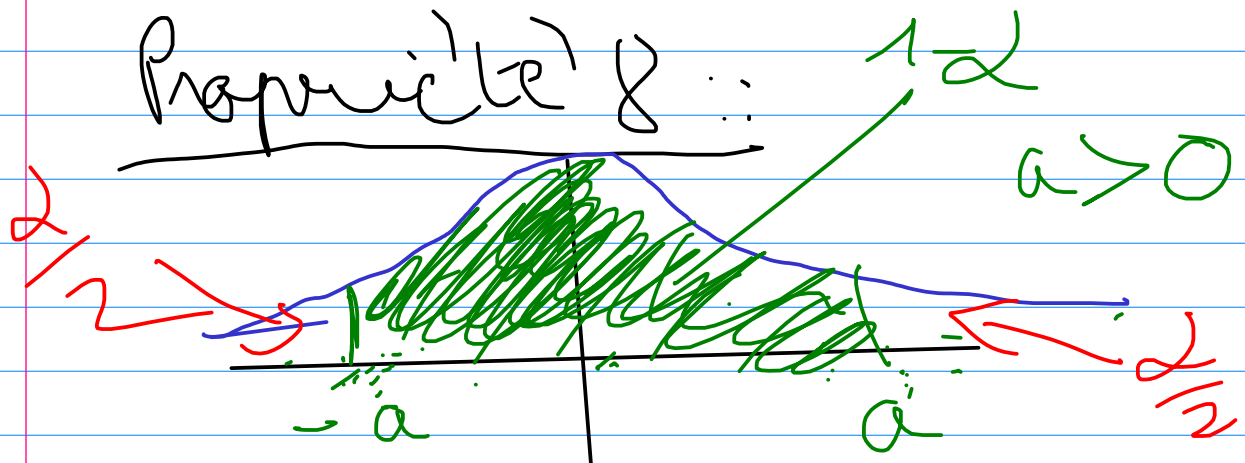
Si $a > 0$ alors $\alpha \geq \frac{1}{2}$

Propriété : Pour tout $\alpha \in [0;1]$, il existe un unique a tel que $P(X \leq a) = \alpha$.

$a = \text{FracNormale}(0,1,\alpha)$
$= \text{InvNormal}(\alpha, 1, \alpha)$

Calcul pratique avec la

Propriété 8 :



$P(-a \leq X \leq a) = 1 - \alpha$

a est unique pour

$\lambda \in [0;1[$